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Lesson 18: Four Interesting Transformations of Functions

Student Outcomes

* Students examine that a horizontal translation of the graph of corresponds to changing the equation from to .

Lesson Notes

In Lesson 18, students examine horizontal translations (shifts) in the graph of a function and how they are represented in the equation of the function. Students will contrast the horizontal shift to the vertical shift covered in Lesson 17. They should be able to describe the transformations of the graph associated with the transformation of the function, as well as write the equation of a graph based on the translations (shifts) or vertical scalings (stretches) of another graph whose equation is known.

Classwork

Example 1 (8 minutes)

Students explore that a horizontal translation of the graph of corresponds to changing the equation from to for given values of . As an example of MP.3, consider asking students to make a conjecture about how they believe this placement of will affect the graph.

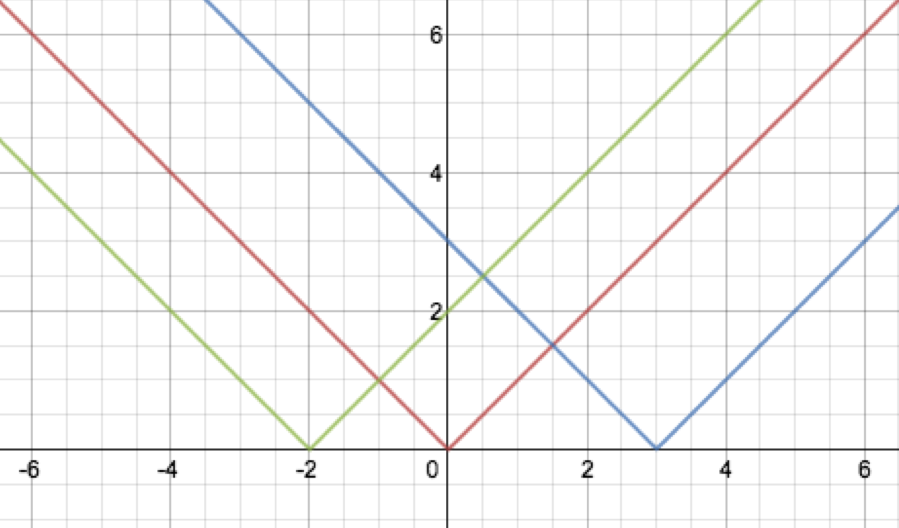
Example 1

Let , , where can be any real number.

* 1. Write the formula for in terms of (i.e., without using notation):
  2. Write the formula for in terms of (i.e., without using notation):
  3. **Complete the table of values for these functions**.

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* 1. G**raph all three equations: , , and** .



* 1. **How does the graph of relate to the graph of ?**

**The graph of is the graph of translated horizontally to the right units.**

* 1. **How does the graph of relate to the graph of ?**

**The graph of is the graph of translated horizontally to the left units.**

* 1. How does the graph of and the graph ofrelate differently to the graph of ?

**The graph of translates the graph of down 3 units whereas the graph of translates the graph of to the right 3 units.**

* 1. **How do the values of and relate to the values of ?**

**The input value for has to be 3 more than the input value for to get the same output values. The input value for has to be two more than the input value for to get the same output values.**

**Discussion (5 minutes)**

*Scaffolding:*

Gives guidance specific to this proportion of the lesson for addressing needs of diverse learners. The type of diverse learner should be specified (i.e., advanced learner, etc.)

Students should finish Example 1 with the understanding that the graph of a function found by subtracting a number to the input of another function, as in , is a translation of the graph of the function horizontally by units (positively or negatively, depending on the sign of ).

* If we replace 3 by a number in as in Example 1 to get , explain how to translate the graph of to the graph of in terms of .
  + *If , then the graph of is translated to the right by units.*
  + *If , then the graph of is translated to the left by units.*
  + *In general, for any , the graph of is translated horizontally by units (where corresponds to a translation to the right and corresponds to a translation to the left).*
* How does your answer for make sense for ?
  + *We can rewrite as . Therefore, since , the graph of should be the translation of the graph of to the left by units.*
* What concept from Grade 8 Geometry best describes the shifts of the graphs of the functions in Example 1?
  + *Translation. In fact, we use the word “translate” to help you remember.*
* Students should be comfortable explaining the difference between the translations of the graphs and .
* Students may confuse the direction of a horizontal translation since the equation may seem to indicate the “opposite” direction (i.e., may be confused as a translation to the right because of the addition of 3 to ), especially since a vertical translation up is the transformation given by adding a positive number to the function. Help students articulate why the horizontal translation behaves as it does.
* Consider the function and its graph from Example 1. There is a point on the graph of . We have . Then the point is on the graph of . Since is on the graph of and is shifted 3 units to the right, we conclude that the graph of is the graph of translated 3 units to the right. A similar argument can be made for the graph of .

Exercises 1–3 (15 minutes)

Have students discuss the following four exercises in pairs. Discuss the answers as a class.

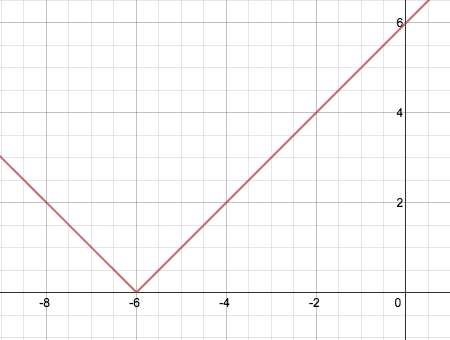
Exercises 1–3

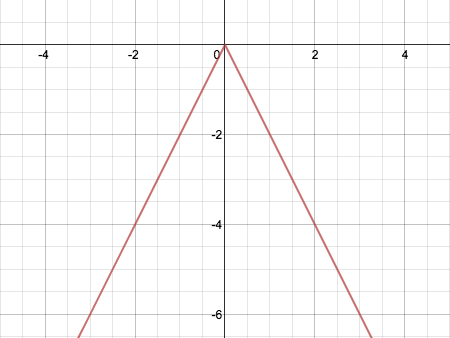
1. Karla and Isamar are disagreeing over which way the graph of the function is translated relative to the graph of . Karla believes the graph of is “to the right “of the graph of , Isamar believes the graph is “to the left.” Who is correct? Use the coordinates of the vertex of and and to support your explanation.

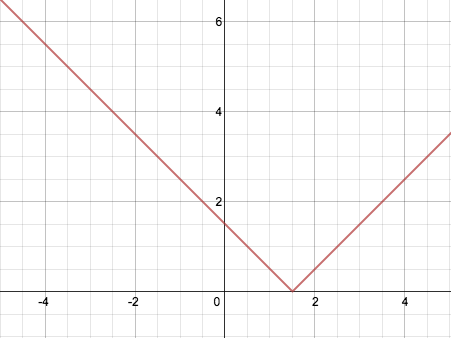
**The graph of is the graph of translated to the left. The vertex of the graph of is the point , whereas the vertex of the graph of is the point .**

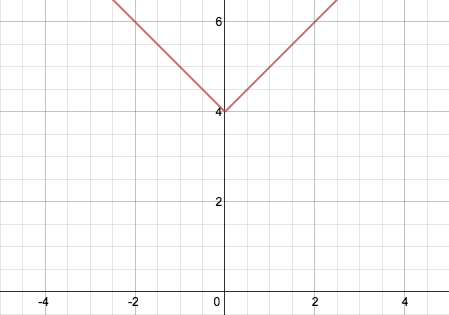
Note that in this lesson, students are working with translations of the function . This function was chosen because it is one of the easier functions to use in showing how translations behave—just follow what happens to the vertex. We know that , or the vertex, is the point of the graph of where the function’s outputs change between decreasing and increasing. As a horizontal translation, the vertex of the graph of will also have a -coordinate of ; in fact, the vertex is Thus the graph of is translated 3 units to the left to get the graph of .

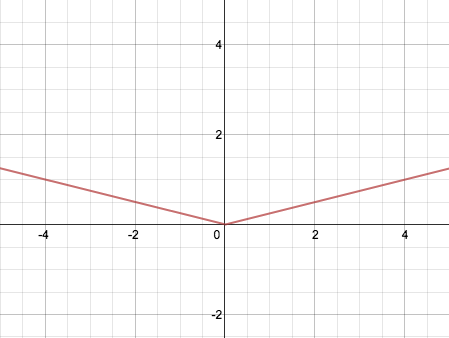
1. Let where can be any real number. Write a formula for the function whose graph is the transformation of the graph of given by the instructions below.
   1. A translation right 5 units.
   2. A translation down 3 units.
   3. A vertical scaling (a vertical stretch) with scale factor of .
   4. A translation left 4 units.
   5. A vertical scaling (a vertical shrink) with scale factor of .
2. Write the formula for the function depicted by the graph.









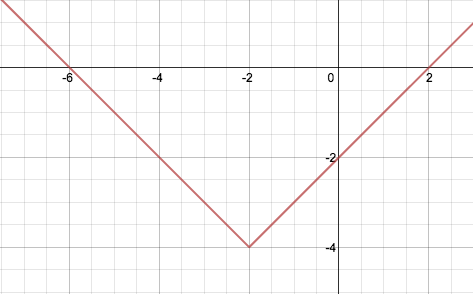


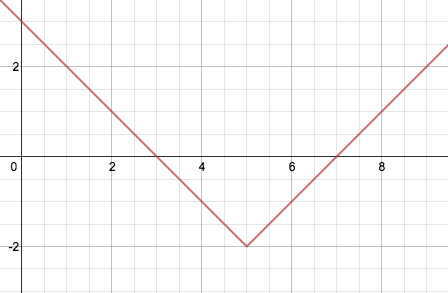
Exercises 4–5 (12 minutes)

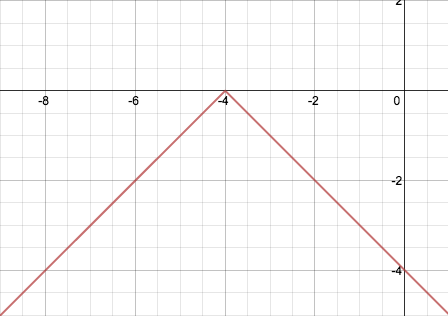
Students now examine questions where more than one change is applied to .

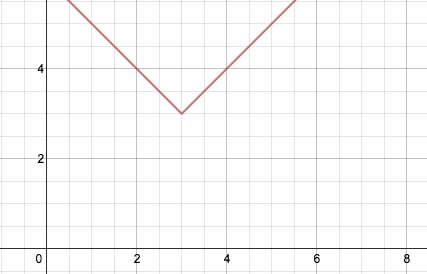
Exercises 4–5

1. Let where can be any real number. Write a formula for the function whose graph is the described transformation of the graph of .
   1. A translation units left and units down.
   2. A translation units right and unit up.
   3. A vertical scaling with scale factor , and then a translation units right.
   4. A translation units right and a vertical scaling by reflected across the -axis with vertical scale factor .
2. Write the formula for the function depicted by the graph.









Closing (2 minutes)

* There is nothing special about using the function as we did in this lesson. The effects of these transformations on the graph of a function hold true for all functions.
* How can the graph of can be horizontally translated by positive or negative ?
* Draw a graph of a made-up function on the board, labeled by , and show how to translate it right or left by units using the equation .

Exit Ticket (3 minutes)

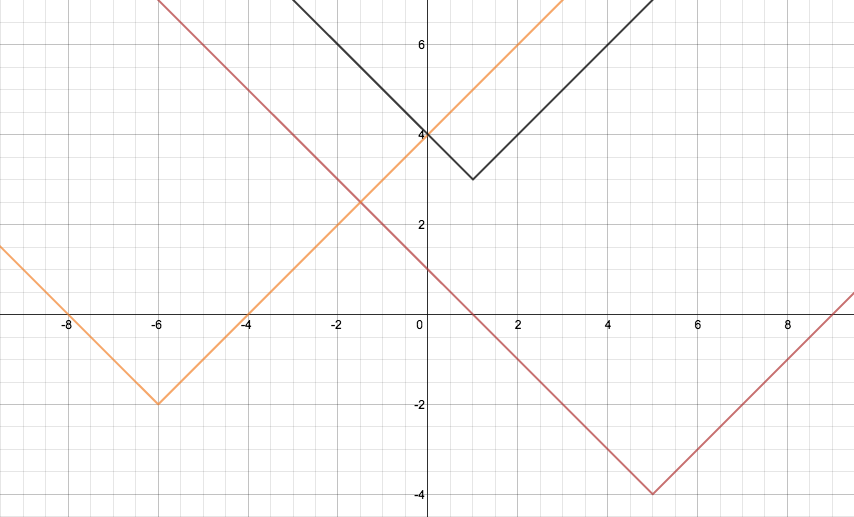
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Lesson 18: Four Interesting Transformations of Functions

Exit Ticket

Write the formula for the functions depicted by the graphs below:

* 1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
  3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Exit Ticket Sample Solutions

Write the formula for the functions depicted by the graphs below:



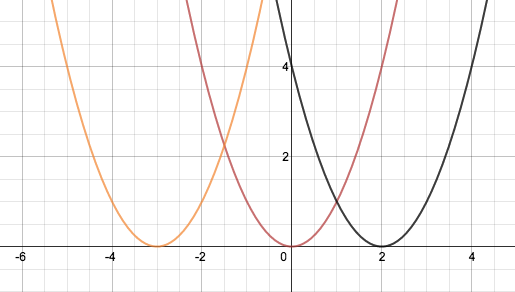
Problem Set Sample Solutions

1. Working with quadratic functions.
   1. The vertex of the quadratic function is at , which is the minimum for the graph of Based on your work in this lessons, to where do you predict the vertex will be translated for the graphs of and ?

The vertex of will be at ; The vertex of will be at .

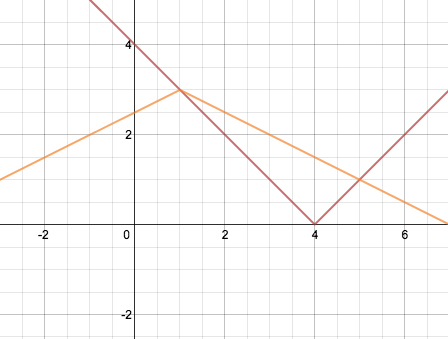
* 1. **Complete the table of values and t**hen g**raph all three functions**.

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1. Let for every real number . The graph of the equation is provided on the Cartesian plane below. Transformations of the graph of are described below. After each description, write the equation for the transformed graph. Then, sketch the graph of the equation you write for part (d).
   1. Translate the graph left units and down units.
   2. Reflect the resulting graph from part (a) across the -axis.

* 1. Scale the resulting graph from part (b) vertically by a scale factor of .
  2. Translate the resulting graph from part (c) right units and up units. Graph the resulting equation.



1. Let for all real numbers . Write the formula for the function represented by the described transformation of the graph of .
   1. First, a vertical stretch with scale factor is performed, then a translation right units, and finally a translation down unit.
   2. First, a vertical stretch with scale factor is performed, then a reflection over the -axis, then a translation left units, and finally a translation up units.
   3. First, a reflection across the -axis is performed, then a translation left 4 units, then a translation up 5 units, and finally a vertical stretch with scale factor 3.

* 1. Compare your answers to parts (b) and (c). Why are they different?

**In part (c), the vertical stretch happens at the end, which means the graph resulting from the first three transformations is what is vertically stretched: .**

1. Write the formula for the function depicted by each graph.

